

Rapid dissipative preparation of weakly interacting real band insulators

Yuxuan Zhang

Institute of Physics, École Polytechnique Fédérale de Lausanne (EPFL)
CH-1015 Lausanne, Switzerland

Department of Physics, Princeton University
Princeton, New Jersey 08544, USA

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Abstract

We study energy-lowering filtered Lindblad dynamics for a finite-range real fermionic hopping Hamiltonian with filled and empty bands separated from zero by a fixed gap. For sufficiently weak bounded finite-range even interactions, the original dynamics with physical annihilation and creation couplings has a unique pure ground-state fixed point and mixes from every even state in time $O(\log(N/\eta))$. The coupling threshold and the implicit constant are independent of the volume. A real particle-hole transformation converts the free dynamics to sitewise amplitude damping while turning the interaction into a uniformly summable quasi-local interaction. We prove stability directly in a fermionic oscillator seminorm, retaining these tails. This supplies the locality and contraction estimates needed beyond the positive-hopping theorem.

1 Setting and theorem

Let Λ be a finite lattice region with N fermionic modes c_i and polynomial ball growth

$$|B(i, r)| \leq C_b(1 + r)^D. \quad (1.1)$$

The geometry, local mode density, interaction ranges, and local norm bounds are fixed across volumes. Consider

$$H_\lambda = \sum_{i,j \in \Lambda} M_{ij} c_i^\dagger c_j + \lambda \sum_X v_X. \quad (1.2)$$

The matrix M is real symmetric, has range at most R_0 , and obeys a uniform absolute row-sum bound J_0 . Its spectrum satisfies

$$\text{spec } M \subset [-J, -\Delta] \cup [\Delta, J], \quad 0 < \Delta \leq J, \quad (1.3)$$

with both bands nonempty. Each v_X is Hermitian and even, supported on a set of bounded diameter, with bounded norm and bounded overlap. Equivalently, one may fix a bound on $\sup_i \sum_{X \ni i} \|v_X\|$ together with the range and local density bounds.

We use the energy-lowering filter and rate convention of Zhan et al. [1]. Fix the filter in terms of J, Δ so that

$$\widehat{f}(\nu) = 1 \quad (-J \leq \nu \leq -\Delta), \quad \widehat{f}(\nu) = 0 \quad (\nu > 0), \quad (1.4)$$

and $|f(t)| \leq C_f e^{-c_f |t|^b}$ for some $0 < b \leq 1$. Here $\widehat{f}(\nu) = \int_{\mathbb{R}} f(t) e^{it\nu} dt$. Since $f \in L^1$, $\widehat{f}$ is continuous and $\widehat{f}(0) = 0$. For every physical coupling $A \in \{c_i, c_i^\dagger : i \in \Lambda\}$, put

$$K_A = \mathcal{F}_{H_\lambda}(A) = \int_{\mathbb{R}} f(t) e^{itH_\lambda} A e^{-itH_\lambda} dt.$$

The Heisenberg generator and its Schrödinger adjoint are

$$\mathcal{L}_\lambda = \sum_A \mathcal{D}(K_A), \quad \mathcal{D}(K)(O) = K^\dagger O K - \frac{1}{2} \{K^\dagger K, O\}, \quad \mathcal{L}_\lambda^*. \quad (1.5)$$

There is no coherent term. In particular, no physical $i[H_\lambda, O]$ is added to (1.5).

A state or observable is *even* if it commutes with total fermion parity.

Theorem 1.1. *There are $\lambda_* > 0$ and $C < \infty$, depending only on the fixed geometry, local bounds, J, Δ , and filter, such that for every allowed volume and $|\lambda| \leq \lambda_*$, the following hold. The Hamiltonian H_λ has a unique ground vector up to phase, denoted Ω_λ . Its projector is the unique stationary density operator of (1.5). For every even initial state ρ and $0 < \eta < 1$,*

$$\frac{1}{2} \left\| e^{t\mathcal{L}_\lambda^*}(\rho) - |\Omega_\lambda\rangle\langle\Omega_\lambda| \right\|_1 \leq \eta \quad \text{whenever} \quad t \geq C \log(N/\eta). \quad (1.6)$$

With the plateau normalized as in (1.4), the proof gives $\frac{1}{2} \|\cdot\|_1 \leq 4Ne^{-t/4}$ and permits $C = 12$ after choosing the perturbative threshold.

Theorem 11 of [1] assumes a positive one-particle matrix. A filled negative band can be exchanged with holes, but that change makes local interactions quasi-local. The proof below controls this change explicitly and then proves contraction for the resulting quasi-local generator. The real-hopping assumption is retained throughout; this manuscript does not claim the corresponding complex-hopping extension.

2 Particle–hole coordinates and uniform locality

Let $P_\pm$ be the spectral projectors of M onto its positive and negative bands. They are real symmetric. Define

$$d = P_+ c + P_- c^\dagger. \quad (2.1)$$

The orthogonality relations for $P_\pm$ imply the CAR $\{d_i, d_j^\dagger\} = \delta_{ij}$ and $\{d_i, d_j\} = 0$. The full Nambu matrix

$$\begin{pmatrix} P_+ & P_- \\ P_- & P_+ \end{pmatrix}$$

is real orthogonal and is its own inverse. Applying the CAR on the negative subspace gives

$$c^\dagger M c = \text{Tr}(M P_-) + d^\dagger |M| d. \quad (2.2)$$

The two total parity operators differ by a constant sign, so their even algebras coincide.

Lemma 2.1. *There are $C, \mu > 0$, independent of the volume, such that*

$$|(P_+)_{ij}| + |(P_-)_{ij}| + |(|M|)_{ij}| \leq Ce^{-\mu \text{dist}(i,j)}.$$

When expressed in the d modes, both $d^\dagger |M| d$ and the perturbation have uniformly bounded exponentially weighted interaction norms. In particular, the transformed perturbation admits even shell terms $\Phi_{X,r}$ supported on $X_r = \{i : \text{dist}(i, X) \leq r\}$ such that, for some $a > 0$,

$$\sup_y \sum_{\substack{X_r \\ y \in X_r}} |X_r| e^{a \text{diam}(X_r)} \|\Phi_{X,r}\| \leq C_V. \quad (2.3)$$

Proof. Choose a contour around one spectral band at distance at least $\delta > 0$ from the spectrum, where δ and the contour length depend only on J, Δ . For a fixed vertex x , conjugate M by the diagonal matrix with entries $e^{\mu \text{dist}(i,x)}$. Finite range and the row and column sum bounds show, by the Schur test, that this changes M in norm by at most $J_0(e^{\mu R_0} - 1)$. Choose μ so that this is at most $\delta/2$. The conjugated resolvent has norm at most $2/\delta$ on the contour. Integrating the resolvent proves the exponential projector bound. The identity $|M| = M(P_+ - P_-)$ gives the same type of bound for $|M|$.

A local algebra on at most s_0 modes is spanned by at most 4^{s_0} Majorana monomials of degree at most $2s_0$. The coefficients in the normalized trace expansion of v_X have modulus at most $\|v_X\|$. Substituting (2.1) into such a monomial gives a product of bounded linear Majorana combinations whose coefficient tails are exponential. Truncating each linear combination to X_r has error at most $Ce^{-\nu r}$ for any fixed $\nu < \mu$; polynomial ball growth absorbs the shell count. A telescoping product estimate therefore produces an even, Hermitian local approximant $v_X^{(r)}$ with

$$\|\tilde{v}_X - v_X^{(r)}\| \leq C \|v_X\| e^{-\nu r}.$$

Hermiticity can also be imposed by averaging the approximant with its adjoint, without increasing the error. Set $\Phi_{X,0} = v_X^{(0)}$ and $\Phi_{X,r} = v_X^{(r)} - v_X^{(r-1)}$ for $r \geq 1$.

At radius r , both $|X_r|$ and the number of original bounded-size sets X whose neighborhood contains a fixed y have polynomial growth. Also $\text{diam}(X_r) \leq R_V + 2r$. Thus (2.3) is bounded by a fixed constant times

$$\sum_{r \geq 0} (1+r)^{2D} e^{-(\nu-2a)r},$$

which converges when $2a < \nu$. The quadratic term is controlled by the exponential entries of $|M|$ and the same counting argument. $\square$

The weighted bound supplies the fermionic Lieb–Robinson assumptions for an even quasi-local interaction [2, Theorem 3.1]. For example, a decay function $e^{-a'r}/(1+r)^{D+1}$ with $a' < a$ absorbs the remaining polynomial factors and has a volume-independent convolution bound.

Filtering is linear in its coupling operator, and a unitary mixing of all jump labels preserves the sum of dissipators. The Nambu matrix in (2.1) is orthogonal. Hence the *same* physical generator (1.5) can be written using the filtered labels $d_i, d_i^\dagger$. At $\lambda = 0$, (1.4) and (2.2) give

$$K_i^-(0) = d_i, \quad K_i^+(0) = 0, \quad \mathcal{L}_0 = \sum_i \mathcal{D}(d_i). \quad (2.4)$$

No additional choice of localized one-particle eigenvectors is needed.

3 Local filtered jumps and small perturbations

All support statements in this section refer to the CAR algebra of the d modes. Write $\tau_t^\lambda(O) = e^{itH_\lambda} O e^{-itH_\lambda}$.

Lemma 3.1. *The filtered jumps admit odd approximants $K_i^{\sigma,r}(\lambda)$ supported on $B(i, r)$ such that*

$$\|K_i^\sigma(\lambda) - K_i^{\sigma,r}(\lambda)\| \leq g(r), \quad g(r) \leq C e^{-cr^b}, \quad (3.1)$$

$$\sup_{i,\sigma} \|K_i^\sigma(\lambda) - K_i^\sigma(0)\| \leq \eta(\lambda), \quad \eta(\lambda) \rightarrow 0 \quad (\lambda \rightarrow 0). \quad (3.2)$$

The constants are uniform in the volume for $|\lambda| \leq 1$. The approximating maps are contractions and preserve d_i and zero.

Proof. Take a Majorana γ_i at site i . The operator $K_i^\sigma \gamma_i$ is even. Let $\mathbb{E}_{B(i,r)}$ be the parity-compatible conditional expectation onto the ball used in the fermionic localization lemma [2, Section 4], and define

$$K_i^{\sigma,r} = \mathbb{E}_{B(i,r)}(K_i^\sigma \gamma_i) \gamma_i. \quad (3.3)$$

The conditional expectation is applied to an even operator, so the cited localization result applies in its stated domain. The final product is odd and local, and contractivity follows since γ_i is unitary.

To bound the localization error before filtering, test $\tau_t^\lambda(d_i) \gamma_i$ against an exterior operator of definite parity. For an exterior even operator, the commutator reduces to the ordinary Lieb–Robinson commutator of $\tau_t^\lambda(d_i)$. For an exterior odd operator B ,

$$[\tau_t^\lambda(d_i) \gamma_i, B] = -\{\tau_t^\lambda(d_i), B\} \gamma_i.$$

The graded Lieb–Robinson bound controls this anticommutator. These two parity cases cover all exterior operators, as required for conditional-expectation localization. Summing the spatial decay over the exterior gives a bound of the form $C e^{v|t| - ar}$ up to polynomial factors.

Split the filter integral at $|t| = \theta r$ with $\theta > 0$ small enough that $v\theta < a/2$. The inner part is exponentially small in r ; the outer part is bounded using $\int_{|t| > \theta r} |f(t)| dt$. Absorbing polynomials into a smaller exponent gives (3.1).

For the coupling change, Duhamel’s formula and the local interaction norm yield

$$\left\| \tau_t^\lambda(A_i) - \tau_t^0(A_i) \right\| \leq C |\lambda| |t| e^{v|t|}, \quad A_i = d_i, d_i^\dagger.$$

The sum of commutators with interaction terms is controlled by the spatial Lieb–Robinson sum, not by $\|\sum_X v_X\|$. Integrating first over $|t| \leq T$ gives

$$\eta(\lambda) \leq C |\lambda| T e^{vT} + 2 \int_{|t| > T} |f(t)| dt. \quad (3.4)$$

One can take $T = \log(1/|\lambda|)/(2v)$ when $v > 0$, or first choose T large and then λ small. This proves (3.2) uniformly. $\square$

Let $\mathcal{L}_i(\lambda)$ be the sum of the two exact site-labelled dissipators. Set

$$\mathcal{A}_i^r = \mathcal{D}(K_i^{-,r}) + \mathcal{D}(K_i^{+,r}), \quad \mathcal{E}_{i,0} = \mathcal{A}_i^0 - \mathcal{D}(d_i), \quad \mathcal{E}_{i,r} = \mathcal{A}_i^r - \mathcal{A}_i^{r-1} \quad (r \geq 1).$$

Then

$$\mathcal{L}_i(\lambda) - \mathcal{D}(d_i) = \sum_{r \geq 0} \mathcal{E}_{i,r}. \quad (3.5)$$

Every shell is local on $X_{i,r} = B(i, r)$, preserves total parity, and annihilates the identity. In the norm induced by the full operator norm,

$$\|\mathcal{E}_{i,r}\| \leq \epsilon_r(\lambda), \quad \epsilon_0 = C\eta(\lambda), \quad \epsilon_r = C \min\{\eta(\lambda), g(r-1)\} \quad (r \geq 1). \quad (3.6)$$

This follows from $\|\mathcal{D}(K) - \mathcal{D}(L)\| \leq 2(\|K\| + \|L\|)\|K - L\|$, contractivity of the localizing maps, and (2.4). In particular,

$$\sum_{r \geq 0} (1+r)^{2D} \epsilon_r(\lambda) \longrightarrow 0. \quad (3.7)$$

4 Oscillations on the even algebra

Let $\mathcal{T}_i$ be the normalized tracial conditional expectation onto the CAR algebra of the complement of site i . On even observables these expectations commute and are contractions. Put

$$\delta_i = \text{id} - \mathcal{T}_i, \quad Z_i = 1 - 2d_i^\dagger d_i, \quad \mathcal{P}_i(O) = \frac{1}{2}(O + Z_i O Z_i), \quad \mathcal{Q}_i = \text{id} - \mathcal{P}_i.$$

Both $\mathcal{P}_i$ and $\mathcal{Q}_i$ have operator norm at most one. Define

$$\Delta_i^+ = \delta_i \mathcal{P}_i, \quad \Delta_i^- = \delta_i \mathcal{Q}_i, \quad s_i(O) = \|\Delta_i^+ O\| + \|\Delta_i^- O\|, \quad \mathcal{S}(O) = \sum_i s_i(O). \quad (4.1)$$

The definitions are the fermionic oscillator interface underlying [1, Appendix I]. Telescoping the commuting expectations, and using $\delta_i = \Delta_i^+ + \Delta_i^-$, gives

$$\|O - \text{tr}(O)I\| \leq \mathcal{S}(O), \quad \mathcal{S}(O) \leq 4N \|O\|, \quad \text{tr}(O) = 2^{-N} \text{Tr } O. \quad (4.2)$$

Lemma 4.1. *Let $\mathcal{E}_X$ be a local superoperator on X , expressible as a sum of terms pOq of total even parity, and suppose $\mathcal{E}_X(I) = 0$. For even O ,*

$$\mathcal{E}_X \mathcal{T}_X O = 0, \quad \mathcal{T}_X = \prod_{j \in X} \mathcal{T}_j, \quad (4.3)$$

$$[\Delta_i^\sigma, \mathcal{E}_X]O = 0 \quad (i \notin X), \quad (4.4)$$

$$\|\mathcal{E}_X O\| \leq \|\mathcal{E}_X\| \sum_{j \in X} s_j(O). \quad (4.5)$$

For $i \in X$,

$$\|\Delta_i^\sigma \mathcal{E}_X O\| \leq 2 \|\mathcal{E}_X\| \sum_{j \in X} s_j(O), \quad (4.6)$$

$$\|[\Delta_i^\sigma, \mathcal{E}_X]O\| \leq 3 \|\mathcal{E}_X\| \sum_{j \in X} s_j(O). \quad (4.7)$$

Proof. The observable $\mathcal{T}_X O$ is even and supported outside X , so it commutes with all local coefficients in $\mathcal{E}_X$. Therefore $\mathcal{E}_X(\mathcal{T}_X O) = \mathcal{T}_X O \mathcal{E}_X(I) = 0$. The CAR and the tracial expectation give (4.4); equivalently, this can be checked on Majorana monomials. If a monomial contains a Majorana at $i \notin X$, multiplication by a coefficient supported on X cannot remove that factor; if it does not, the expectation at i is unchanged. The local parity map at i commutes as well.

Telescoping $O - \mathcal{T}_X O$ bounds its norm by $\sum_{j \in X} s_j(O)$, proving (4.5). Since $\|\Delta_i^\sigma\| \leq 2$, the first inside bound follows. For the commutator, its first term has that bound and its second has norm at most $\|\mathcal{E}_X\| \|\Delta_i^\sigma O\| \leq \|\mathcal{E}_X\| s_i(O)$. $\square$

The identity-annihilation assumption in this lemma is essential. It is satisfied by every shell in (3.5); locality and parity alone would not imply (4.3).

For the free one-site dissipator, direct CAR calculation yields

$$\Delta_i^+ \mathcal{D}(d_i)O = -\Delta_i^+ O, \quad \Delta_i^- \mathcal{D}(d_i)O = -\frac{1}{2}\Delta_i^- O \quad (4.8)$$

on the even algebra. One may verify these identities by separating local-even monomials, generated by I, Z_i , from local-odd monomials paired with odd exterior monomials. This is an exact algebraic identity, not an estimate from a spectral gap.

5 Contraction and identification of the stationary state

Write $\mathcal{E}_i = \sum_r \mathcal{E}_{i,r}$ and $O(t) = e^{t\mathcal{L}_\lambda} O$. Applying Δ_i^σ and commuting it through all site terms except the free term at i gives

$$\frac{d}{dt} \Delta_i^\sigma O(t) = \left(\sum_{j \neq i} \mathcal{L}_j(\lambda) - a_\sigma \right) \Delta_i^\sigma O(t) + F_i^\sigma(t), \quad (5.1)$$

$$F_i^\sigma(t) = \Delta_i^\sigma \mathcal{E}_i O(t) + \sum_{j \neq i} [\Delta_i^\sigma, \mathcal{E}_j] O(t), \quad a_+ = 1, \quad a_- = \frac{1}{2}. \quad (5.2)$$

The homogeneous generator uses the *actual* sum of all site-labelled Lindblad terms except i . Its semigroup is unital completely positive and therefore contractive. This prevents an extensive perturbation from entering the homogeneous bound.

Lemma 4.1, summed over sites and both signs, gives

$$\sum_{i,\sigma} \|F_i^\sigma(t)\| \leq \kappa(\lambda) \mathcal{S}(O(t)), \quad \kappa(\lambda) = 6C_b^2 \sum_{r \geq 0} (1+r)^{2D} \epsilon_r(\lambda). \quad (5.3)$$

One ball-volume factor counts the sites i affected by a shell centered at j ; the other counts centers j whose ball contains a fixed derivative site. Both are uniform by (1.1). Duhamel's formula then yields

$$\mathcal{S}(O(t)) \leq e^{-t/2} \mathcal{S}(O) + \kappa(\lambda) \int_0^t e^{-(t-u)/2} \mathcal{S}(O(u)) du.$$

By (3.7), choose $\lambda_* > 0$ with $\kappa(\lambda) \leq 1/4$ for all $|\lambda| \leq \lambda_*$. This can be done by first fixing a radius whose weighted tail contributes less than $1/8$, and then making the finite initial sum less than $1/8$. Gronwall's inequality gives

$$\mathcal{S}(e^{t\mathcal{L}_\lambda} O) \leq e^{-t/4} \mathcal{S}(O). \quad (5.4)$$

Every ground vector is dark for every filtered jump: a matrix element from a ground vector has nonnegative Bohr frequency and is annihilated by (1.4), including frequency zero. Thus every ground-state projector is stationary. Since H_λ is even, its ground space has a parity eigenbasis.

For even states ρ, σ , trace duality can be restricted to even observables. Equations (4.2) and (5.4) give

$$\frac{1}{2} \|e^{t\mathcal{L}_\lambda^*}(\rho - \sigma)\|_1 \leq 4Ne^{-t/4}. \quad (5.5)$$

Indeed, subtracting $\text{tr}(e^{t\mathcal{L}_\lambda} O)I$ in the dual pairing costs no scalar term, and $\|\rho - \sigma\|_1 \leq 2$. Therefore there is at most one even stationary density. Distinct parity eigenvectors in the ground space would have distinct even stationary projectors, so the ground space is one dimensional.

Finally, parity twirling any stationary density produces this pure even stationary density. A pure state is an extreme point of the convex state space; if it is the average of a state and its parity conjugate, both states equal it. Thus the stationary density is unique on the full state space. Taking $\sigma = |\Omega_\lambda\rangle\langle\Omega_\lambda|$ in (5.5) proves the asserted mixing estimate. Both free bands are nonempty, so $N \geq 2$, and

$$4 \log(4N/\eta) \leq 12 \log(N/\eta), \quad 0 < \eta < 1.$$

This completes the proof of Theorem 1.1.

6 Verification and scope

The companion archive records the fixed original statement, candidate M03-v3, the locality-interface derivation, and the subsequent strict audit. Finite CAR controls check the free identities and exterior-locality relations on a basis of the four-mode even algebra. These checks are useful for signs and conventions; the volume-independent result follows from Lemmas 2.1–4.1 and the summable bound (5.3).

The result is an upper bound for the physical, unrescaled jump list and the specified fixed filter. Its constants may depend on the gap, bandwidth, geometry, and local strengths. No claim of uniformity as $\Delta \downarrow 0$, a matching mixing lower bound, or a complex-hopping theorem is made. The filtered preparation construction and fermionic oscillator interface are inherited from [1]; the argument above makes explicit the quasi-local filled-band reduction and its stable contraction. The proof and closest-source comparisons have received separate-context internal review. This bounded source comparison does not establish exhaustive priority, and external expert review remains outstanding.

AI-assisted research

This work was developed in an author-directed, AI-assisted research workflow. AI systems assisted with mathematical derivation, literature checks, computational verification, and manuscript drafting. The internal reviews and separate implementations described above were produced within this workflow and do not constitute external expert review.

References

- [1] Y. Zhan, Z. Ding, J. Huhn, J. Gray, J. Preskill, G. K.-L. Chan, and L. Lin, *Rapid quantum ground state preparation via dissipative dynamics*, Phys. Rev. X **16**, 011004 (2026). [arXiv:2503.15827v3](#), especially Theorem 11 and Appendix I.
- [2] B. Nachtergaele, R. Sims, and A. Young, *Lieb–Robinson bounds, the spectral flow, and stability of the spectral gap for lattice fermion systems*, [arXiv:1705.08553v3](#), especially Theorem 3.1 and Section 4.