

Two-pass local measurements beat the measured Chernoff exponent

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Abstract

Can classical communication improve the exponential rate of quantum state discrimination when every operation acts on just one input copy? Single-visit adaptive measurements have the same optimal Chernoff exponent as a fixed single-copy measurement. We show that revisiting the copies changes the answer. An explicit pair of faithful, noncommuting four-dimensional states admits a two-pass local protocol with an error exponent strictly larger than the measured Chernoff exponent. The first pass reads an orthogonal flag while retaining a qubit at each party. After the flags are communicated, their majority determines the measurement basis for the second pass. We bound every single-copy POVM analytically and determine the exact exponent of the displayed protocol, with matching bounds valid for every number of copies. For the chosen pair, the measured exponent is below 0.677144, whereas the protocol exponent exceeds 0.68250256. The construction uses classical communication and local quantum storage, without shared entanglement or a joint operation on two input copies. It gives a strict asymptotic separation from single-visit measurements and disproves the proposed universal equality of the LOCC, separable, and measured Chernoff exponents. The optimal unrestricted LOCC and separable exponents remain undetermined.

1 Introduction

Repeated preparations make two imperfectly distinguishable quantum states increasingly easy to tell apart. With equal prior probabilities, the optimal error often decays as $\exp(-n\xi)$, where n is the number of preparations and ξ is an error exponent. The quantum Chernoff theorem determines this exponent when arbitrary collective measurements are available [1, 2]. Local measurements pose a different question: how much of the collective advantage can be recovered by exchanging only classical information?

There are two ways to impose locality across the copies. A party can finish measuring its copy and pass the classical result to the next party. Alternatively, it can make a partial measurement, retain its remaining quantum state, and return to it after learning what the other parties observed. Hayashi proved that adaptation in the first, single-visit model does not improve the optimal Chernoff exponent over a fixed single-copy measurement [3, Section V]. His concluding discussion leaves open whether two-way LOCC improves asymptotic discrimination beyond the Stein setting [3, Section IX]. The distinction matters because a local instrument can reveal classical information without exhausting the quantum information in its input.

We give a strict separation using two passes through the parties. Each four-dimensional input consists of an orthogonal flag and a conditional qubit. The flag is biased in opposite directions

under the two hypotheses. Reading it leaves the qubit intact, so all parties can first contribute to a shared classical assessment of the hypothesis. They then choose the qubit measurement bases using that assessment. No operation couples different input copies.

The protocol's error is controlled by flag strings close to a tie. On those strings, the two passes combine the classical flag exponent with a residual qubit exponent. The proof compares this performance with a bound that holds uniformly over every single-copy POVM and every Chernoff parameter. It therefore avoids an optimization over a chosen family of measurements. We also identify the exact exponent of the displayed protocol by proving upper and lower error bounds for every n .

The result answers the qualitative improvement question in Hayashi's copywise setting. It does not determine the optimal two-way LOCC exponent. Since every finite-round LOCC test has separable effects, the same example also rules out a universal equality between the separable exponent and the measured exponent. Whether unrestricted LOCC and separable tests have the same optimal exponent remains a separate question.

Related work. Calsamiglia, de Vicente, Muñoz-Tapia and Bagan studied mixed-qubit discrimination using finite-copy partial-transpose bounds and adaptive strategies [4]. Their asymptotic fits for $25 \leq n \leq 35$ are consistent with the measured exponent for the examples studied. Our states have dimension four, and the separation below follows from bounds valid for all n . Matthews and Winter considered a different locality constraint: two laboratories holding many copies of a bipartite state, with collective operations allowed within each laboratory [5]. Here there are n parties, each holding one whole input copy. Results on adaptive channel discrimination also use a different operational interface; for example, the receiver in Salek, Hayashi and Winter may retain quantum channel outputs for a final joint measurement [6, Section II]. Our comparison is with a single-copy measured exponent, and all input copies remain at separate parties. The measured-coefficient estimate used below follows the variational approach to measured Rényi quantities developed in [7]; we include the needed inequality and its proof.

2 Measurement model and main result

All logarithms are natural. Let $\rho, \sigma > 0$ be density matrices on a finite-dimensional Hilbert space $\mathcal{H}$. Under the two equally likely hypotheses, n parties receive $\rho^{\otimes n}$ or $\sigma^{\otimes n}$, with one tensor factor per party. A binary test $\{T, I - T\}$ has error

$$p_e(T; n) = \frac{1}{2} [\text{Tr } \rho^{\otimes n}(I - T) + \text{Tr } \sigma^{\otimes n}T]. \quad (1)$$

The class LOCC_n consists of tests exactly implemented by a finite number of local operations and classical messages. Parties may be revisited; the number of operations may depend on n . Local quantum storage is allowed. There is no shared entangled resource, quantum communication, or operation on two input copies. The class SEP_n consists of binary tests for which both effects are fully separable across the n parties. In particular, $\text{LOCC}_n \subseteq \text{SEP}_n$.

For either class $\mathcal{C}$, write $p_e^{\mathcal{C}}(n) = \inf_{T \in \mathcal{C}_n} p_e(T; n)$ and define

$$\underline{\xi}_{\mathcal{C}} = \liminf_{n \rightarrow \infty} -\frac{1}{n} \log p_e^{\mathcal{C}}(n), \quad \bar{\xi}_{\mathcal{C}} = \limsup_{n \rightarrow \infty} -\frac{1}{n} \log p_e^{\mathcal{C}}(n). \quad (2)$$

This notation makes no assumption that an optimal-class exponent converges. For a finite-outcome

single-copy POVM $M = \{M_i\}$, put $P_\tau^M(i) = \text{Tr } \tau M_i$ and

$$C_M(\rho, \sigma) = \sup_M \max_{0 \leq s \leq 1} \left[-\log \sum_i P_\rho^M(i)^{1-s} P_\sigma^M(i)^s \right]. \quad (3)$$

Repeating a fixed measurement and making the optimal classical decision achieves the associated classical Chernoff exponent. The supremum in (3) is the single-visit adaptive benchmark in [3, Section V].

Set

$$\begin{aligned} \varepsilon &= 2^{-16}, & a &= 1 - \varepsilon, & p &= \frac{4}{5}, & A &= \begin{pmatrix} a & 0 \\ 0 & \varepsilon \end{pmatrix}, \\ H &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, & B &= HAH, \\ \rho &= pA \oplus (1-p)B, & \sigma &= (1-p)B \oplus pA. \end{aligned} \quad (4)$$

For $0 < u < 1$, let $h_2(u) = -u \log u - (1-u) \log(1-u)$.

Theorem 1. *The states in (4) are faithful and noncommuting. Their measured Chernoff exponent obeys*

$$C_M(\rho, \sigma) < \log \frac{5}{4} + \frac{454}{1000} < 0.677144. \quad (5)$$

There is a two-pass finite-round LOCC protocol with error $p_e^{\text{tp}}(n)$ satisfying, for every integer $n \geq 1$,

$$\frac{e^{-d-nE}}{4(n+1)^2} \leq p_e^{\text{tp}}(n) \leq \frac{n+1}{2} e^{-nE}, \quad (6)$$

where

$$d = \log \frac{a}{\varepsilon} = \log 65535, \quad u = \frac{\log(2a)}{d}, \quad E = \log \frac{5}{4} + \log 2 - h_2(u). \quad (7)$$

In particular, the protocol has an ordinary error exponent equal to E , and

$$\xi_{\text{SEP}} \geq \xi_{\text{LOCC}} \geq E > C_M(\rho, \sigma) + \frac{3}{1000}. \quad (8)$$

An outward-rounded evaluation of (7) gives $0.68250256 < E < 0.68250257$.

The theorem disproves the universal equality $\xi_{\text{LOCC}} = \bar{\xi}_{\text{LOCC}} = \xi_{\text{SEP}} = \bar{\xi}_{\text{SEP}} = C_M$ posed in the QIQCOP entry [8]. The value E is the exact exponent of the displayed protocol; it is a lower bound on the optimal-class exponents.

3 A uniform bound on single-copy measurements

The upper bound on C_M must cover arbitrary POVMs, including ones that do not first read the direct-sum flag. We begin with a coefficient estimate that makes this possible.

Lemma 2. *Let $X, Y > 0$ be density matrices, $1/2 \leq \alpha < 1$, and $\beta = (1-\alpha)/\alpha$. Every finite POVM $\{M_i\}$ satisfies*

$$\sum_i (\text{Tr } X M_i)^\alpha (\text{Tr } Y M_i)^{1-\alpha} \geq (\text{Tr } X Y^\beta)^\alpha. \quad (9)$$

Proof. Omit zero effects and write $x_i = \text{Tr } XM_i$, $y_i = \text{Tr } YM_i$. Scalar minimization gives

$$\sum_i x_i^\alpha y_i^{1-\alpha} = \inf_{t_i > 0} \left[\alpha \sum_i x_i t_i^{-\beta} + (1-\alpha) \sum_i y_i t_i \right].$$

The minimizing values are $t_i = (x_i/y_i)^\alpha$. For a fixed choice, let $G = \sum_i t_i M_i > 0$ and $v = \text{Tr } YG > 0$. Operator Jensen's inequality for $z^{-\beta}$ gives

$$\sum_i t_i^{-\beta} M_i \geq G^{-\beta}.$$

Also $G^{1/2} Y G^{1/2} \leq vI$, so $Y \leq vG^{-1}$. The Loewner–Heinz inequality, valid here because $0 \leq \beta \leq 1$, yields $Y^\beta \leq v^\beta G^{-\beta}$. Thus, with $c = \text{Tr } XY^\beta$, the expression being minimized is at least

$$\alpha c v^{-\beta} + (1-\alpha)v \geq c^\alpha.$$

The last scalar minimum occurs at $v = c^\alpha$. This proves (9). Appendix A recalls the operator inequalities used in this argument. $\square$

Let Π_0, Π_1 project onto the two summands of $\mathcal{H} = \mathbb{C}^2 \oplus \mathbb{C}^2$. Refining an arbitrary POVM effect M_i into $\Pi_j M_i \Pi_j$ preserves its total probabilities on the block-diagonal states. For $0 < s < 1$, scalar Hölder gives

$$\sum_j x_j^{1-s} y_j^s \leq \left(\sum_j x_j \right)^{1-s} \left(\sum_j y_j \right)^s.$$

Consequently, refinement can only decrease the Chernoff coefficient. The effects restricted to either summand form a qubit POVM.

For $\alpha = \max(s, 1-s)$ and $\beta = (1-\alpha)/\alpha$, apply Lemma 2 in both blocks, interchanging the two hypotheses when needed. Since H interchanges A and B ,

$$\text{Tr } AB^\beta = \text{Tr } BA^\beta = \frac{a^\beta + \varepsilon^\beta}{2}.$$

Every original POVM coefficient is therefore at least

$$\frac{4}{5} \cosh((2\alpha-1)\log 2) \left(\frac{a^\beta + \varepsilon^\beta}{2} \right)^\alpha. \quad (10)$$

This step uses no restriction on the number or ranks of the original effects.

Lemma 3. *For the states (4), the logarithm of (10) is strictly greater than $\log(4/5) - 454/1000$, uniformly for $1/2 \leq \alpha < 1$.*

Proof. Write $L = \log 2$. Since $a^\beta + \varepsilon^\beta \geq a^\beta(1 + \varepsilon^\beta)$, the logarithm in question is at least

$$\log \frac{4}{5} + g(\alpha) + (1-\alpha) \log a, \quad (11)$$

where

$$g(\alpha) = \log \cosh((2\alpha-1)L) - \alpha L + \alpha \log(1 + e^{-16L(1-\alpha)/\alpha}).$$

The last term is a convex perspective of $z \mapsto \log(1 + e^z)$, evaluated on an affine function of α . The first term has second derivative

$$4L^2 \text{sech}^2((2\alpha-1)L) \geq \frac{64}{25} L^2 > \frac{6}{5}$$

on $[1/2, 1]$. Thus $g''(\alpha) > 6/5$.

At $\alpha = 3/4$, put $x = 2^{-16/3}$. Direct differentiation gives

$$\begin{aligned} g(3/4) &= \log 3 - \frac{9}{4}L + \frac{3}{4}\log(1+x), \\ g'(3/4) &= -\frac{1}{3}L + \log(1+x) + \frac{64L}{3} \frac{x}{1+x}. \end{aligned}$$

The rational enclosures

$$\frac{6931}{10000} < L < \frac{6932}{10000}, \quad \log 3 > \frac{10986}{10000}, \quad \frac{248}{10000} < x < \frac{249}{10000} \quad (12)$$

together with $x - x^2/2 \leq \log(1+x) \leq x$ imply

$$g(3/4) > -\frac{443}{1000}, \quad 0 < g'(3/4) < \frac{77}{500}.$$

Strong convexity and completion of the square now give, throughout $[1/2, 1]$,

$$g(\alpha) > -\frac{443}{1000} - \frac{(77/500)^2}{12/5}.$$

Finally, $\log a \geq -\varepsilon/(1-\varepsilon) = -1/65535$ and $1-\alpha \leq 1/2$. Therefore

$$g(\alpha) + (1-\alpha)\log a > -\delta, \quad \delta = \frac{443}{1000} + \frac{(77/500)^2}{12/5} + \frac{1}{131070} < \frac{454}{1000}. \quad (13)$$

The margins are rational; Appendix B gives a direct certificate for (12) and the displayed comparisons. $\square$

Equations (10)–(13) give a common coefficient lower bound $(4/5)e^{-\delta}$ for every POVM and every interior Chernoff parameter. At $s = 0, 1$ the coefficient equals one because the states are faithful. Taking the infimum over both the measurement and the parameter proves (5).

4 The two-pass protocol

Identify each direct summand with a qubit, and let $V_j : \mathcal{H} \rightarrow \mathbb{C}^2$ extract summand j . The first local instrument consists of

$$\mathcal{I}_j(X) = V_j X V_j^*, \quad j = 0, 1.$$

Its sum is trace preserving. The classical outcome j is the flag, and the quantum output is the retained qubit. Their laws are

	$j = 0$	$j = 1$
ρ	p, A	$1-p, B$
σ	$1-p, B$	p, A .

First apply this instrument to every input copy and communicate all flags. Let k be the number of zero flags. If $k \geq n/2$, each party measures in the eigenbasis of its conditional state under ρ : the A basis for flag zero and the B basis for flag one. If $k < n/2$, it uses the corresponding σ eigenbasis. In either basis, order the outcomes with the larger eigenvalue first. Finish with the optimal classical likelihood decision using the complete transcript.

Put $P = (a, \varepsilon)$ and $Q = (1/2, 1/2)$. Measuring A in the B basis, or B in the A basis, gives Q ; measuring either in its own basis gives P . Hence, conditional on any particular flag string, the second-pass laws are $P^{\otimes n}, Q^{\otimes n}$ in the majority-zero branch, and $Q^{\otimes n}, P^{\otimes n}$ in the other branch. The weights of a string with k zeros are

$$w_k = p^k(1-p)^{n-k}, \quad v_k = (1-p)^k p^{n-k}. \quad (14)$$

This is an exact finite-round LOCC implementation. Both passes may use a fixed visiting order. Each party retains one qubit between the passes and is measured once in the second pass. A pass denotes a visit to every party, rather than a single individual LOCC operation. Every terminal transcript effect is a product of local positive effects. Grouping transcripts by the final decision makes both binary effects fully separable.

5 Exact error exponent of the protocol

We first solve the residual classical problem. For $0 \leq t \leq 1$, let

$$r(t) = \sum_i P_i^t Q_i^{1-t} = 2^{t-1}(a^t + \varepsilon^t).$$

Its logarithm is strictly convex. At its stationary point t_* , the tilted small-outcome probability equals $u = \log(2a)/d$, since the tilted mean of $\log(P_i/Q_i)$ vanishes. Thus

$$t_* = \frac{\log((1-u)/u)}{d}, \quad -\log r(t_*) = \log 2 - h_2(u) =: C_{\text{cl}}(P, Q). \quad (15)$$

In particular $0 < t_* < 1/2$. To check the interval explicitly, $u < 1/2$ follows from $4a\varepsilon < 1$. Also $\log a \geq -1/65535$ and $d < 16 \log 2$ imply $u > 1/32$. At $t = 0$ the tilted small-outcome probability is $1/2$, whereas at $t = 1/2$ it is $\sqrt{\varepsilon}/(\sqrt{a} + \sqrt{\varepsilon}) < 1/128$. It decreases strictly with t , proving the claim. The identity for C_{cl} follows from $(1-u)\log a + u\log \varepsilon = -\log 2$.

5.1 Upper bound on the error

Fix a flag string in the branch $k \geq n/2$. The classical overlap of its two weighted conditional laws is at most

$$\sum_z \min\{w_k P^{\otimes n}(z), v_k Q^{\otimes n}(z)\} \leq w_k^{t_*} v_k^{1-t_*} r(t_*)^n.$$

For $x = k/n$ the flag-type weight satisfies

$$\binom{n}{k} w_k^{t_*} v_k^{1-t_*} \leq e^{nf(x)},$$

where

$$f(x) = h_2(x) + t_*[x \log p + (1-x) \log(1-p)] + (1-t_*)[x \log(1-p) + (1-x) \log p].$$

Here we used $\binom{n}{k} \leq e^{nh_2(k/n)}$, including the endpoints by continuity. For $x \geq 1/2$,

$$f'(x) = \log \frac{1-x}{x} + (2t_* - 1) \log 4 < 0,$$

and $f(1/2) = \log(4/5)$. In the branch $k < n/2$, reverse the Chernoff powers; the type function is $f(1-x)$ and gives the same bound. Summing the $n+1$ types, including the boundary type once, and inserting the equal-prior factor proves

$$p_e^{\text{tp}}(n) \leq \frac{n+1}{2} \left(\frac{4}{5} r(t_*) \right)^n = \frac{n+1}{2} e^{-nE}.$$

5.2 A matching lower bound

Retain only flag strings with $k = \lfloor n/2 \rfloor$. If n is even, their two weights equal $(2/5)^n$. If n is odd, the weights are $(2/5)^n/2$ and $2(2/5)^n$. Thus for either parity their weighted overlap is at least $(2/5)^n/2$ times

$$S_n = \sum_z \min\{P^{\otimes n}(z), Q^{\otimes n}(z)\}.$$

The central binomial coefficient is at least $2^n/(n+1)$, so

$$p_e^{\text{tp}}(n) \geq \frac{(4/5)^n}{4(n+1)} S_n. \quad (16)$$

This bound also covers the branch in which the two conditional laws are reversed.

Let $R = (1-u, u)$ be the tilted law at t_* . A mode $\ell = \lfloor (n+1)u \rfloor$ of its binomial count satisfies $|\ell - nu| \leq 1$, and its type probability is at least $1/(n+1)$. On that type the log-likelihood ratio is

$$L_z = \log \frac{P^{\otimes n}(z)}{Q^{\otimes n}(z)} = d(nu - \ell), \quad |L_z| \leq d.$$

The exact tilted identities give

$$\begin{aligned} P^{\otimes n}(z) &= r(t_*)^n R^{\otimes n}(z) e^{(1-t_*)L_z}, \\ Q^{\otimes n}(z) &= r(t_*)^n R^{\otimes n}(z) e^{-t_*L_z}. \end{aligned}$$

Their minimum is at least $e^{-d}r(t_*)^n R^{\otimes n}(z)$. Summing this one type proves $S_n \geq e^{-d}r(t_*)^n/(n+1)$. Equation (16) yields the lower bound in (6). The polynomial factors in the two error bounds disappear after taking $-n^{-1} \log$, establishing the exact exponent E .

5.3 Strict separation

The residual exponent has the elementary lower bound

$$\begin{aligned} C_{\text{cl}}(P, Q) &\geq -\log r(1/4) \\ &= \frac{3}{4} \log 2 - \log(a^{1/4} + 1/16) \\ &> \frac{3}{4} \log 2 - \log(17/16) > \frac{3}{4} \frac{6931}{10000} - \frac{1}{16} > \frac{457}{1000}. \end{aligned}$$

Combining this with (5) gives $E - C_M > 3/1000$. Optimal LOCC and separable errors cannot exceed the displayed protocol's error, which proves (8). Finally, both states have trace one and minimum eigenvalue $\varepsilon/5 = 1/327680$. The off-diagonal entry of $[A, B]$ is $(a - \varepsilon)^2/2 \neq 0$, so $[\rho, \sigma] \neq 0$. This completes the proof of Theorem 1.

6 Why the second pass helps

The flag and the retained qubit play different roles. The flags produce a common classical assessment of the hypothesis. The qubits then provide a measurement whose more informative direction is chosen using that assessment. A single-copy POVM has to choose its treatment of the residual qubit using information from that copy alone. Single-visit adaptation can use earlier copies, but it has already finished measuring them when later flags arrive. The two-pass protocol can use every flag before choosing any of the final qubit bases.

The proof also identifies which records set the error exponent. Typical flag strings strongly favor the correct hypothesis. Strings close to an even split are rarer, but remain ambiguous enough to dominate the asymptotic error. Their flag cost contributes $\log(5/4)$, and the residual discrimination contributes $C_{\text{cl}}(P, Q)$. This explains the additive expression in (7). The matching lower bound shows that this mechanism gives the exact exponent for the protocol.

The strict gap persists for the fixed faithful pair (4); no eigenvalue is taken to zero as n grows. The small fixed value of ε makes the analytic separation convenient. The construction retains n local qubits between the passes. It does not establish a minimum storage requirement, a minimal input dimension for a separation, or an optimal exponent for either unrestricted measurement class.

Reproducibility

The proof above supplies the bounds for all n . The accompanying source package includes a rational certificate for the decisive constants, an independent integer-arithmetic evaluation of the transcript error, and outward-rounded interval evaluation of the exponent and finite- n bounds. These computations check the stated formulas and conventions; finite instances do not establish the asymptotic theorem. The manuscript has undergone internal AI-assisted proof review. External expert confirmation and formal proof-assistant verification have not been obtained.

Author and AI contributions

The author set the research directions, designed the harness that coordinated model tasks, tools, memory, and checking, supplied physical and mathematical intuitions and proof skeletons, and guided the development and revision of the research ideas. AI models developed detailed candidate arguments and derivations, searched the literature, wrote and ran computational checks, and drafted and revised the manuscript under this supervision. K-Dense’s scientific-writing skill [9] assisted manuscript organization and evidence tracking. The author is responsible for the manuscript.

A The operator inequalities in Lemma 2

For completeness, both operator steps can be obtained from inverse order and a resolvent integral. If $Wv = (M_i^{1/2}v)_i$ and $T = \text{diag}(t_i I)$, then $W^*W = I$ and $W^*TW = G$. Compression of the positive block matrix

$$\begin{pmatrix} T & I \\ I & T^{-1} \end{pmatrix} \geq 0$$

by $\text{diag}(W, W)$, followed by a Schur complement, gives $\sum_i t_i^{-1} M_i \geq G^{-1}$. Applying the same argument to $T + \lambda I$ gives $\sum_i (t_i + \lambda)^{-1} M_i \geq (G + \lambda I)^{-1}$. For $0 < \beta < 1$, integrate this inequality against the positive measure $\frac{\sin \pi \beta}{\pi} \lambda^{-\beta} d\lambda$ and use

$$z^{-\beta} = \frac{\sin \pi \beta}{\pi} \int_0^\infty \frac{\lambda^{-\beta}}{z + \lambda} d\lambda.$$

This proves the required Jensen inequality. The case $\beta = 1$ is the inverse inequality itself. Similarly, the representation

$$z^\beta = \frac{\sin \pi \beta}{\pi} \int_0^\infty \lambda^{\beta-1} \frac{z}{z + \lambda} d\lambda$$

and $z(z + \lambda I)^{-1} = I - \lambda(z + \lambda I)^{-1}$ show that $X \leq Y$ implies $X^\beta \leq Y^\beta$. The endpoint cases are immediate. All matrices here are positive definite, so the inverses and the convergent integrals are well defined.

B Rational bounds and numerical evaluation

The logarithm brackets in (12) can be checked without floating-point arithmetic. For $0 < w < 1$,

$$0 < \log \frac{1+w}{1-w} - 2 \sum_{j=0}^{N-1} \frac{w^{2j+1}}{2j+1} \leq \frac{2w^{2N+1}}{(2N+1)(1-w^2)}. \quad (17)$$

Use $(w, N) = (1/3, 12)$ for $\log 2$ and $(1/2, 16)$ for $\log 3$. The bounds on x follow by cubing the rational endpoints and comparing with 2^{-16} . Explicit bounds for the two midpoint quantities are

$$\begin{aligned} g(3/4) &> \frac{10986}{10000} - \frac{9}{4} \frac{6932}{10000} + \frac{3}{4} \left(\frac{248}{10000} - \frac{1}{2} \left(\frac{249}{10000} \right)^2 \right) > -\frac{443}{1000}, \\ g'(3/4) &> -\frac{1}{3} \frac{6932}{10000} + \frac{248}{10000} - \frac{1}{2} \left(\frac{249}{10000} \right)^2 + \frac{64}{3} \frac{6931}{10000} \frac{248/10000}{1 + 249/10000} > 0, \\ g'(3/4) &< -\frac{1}{3} \frac{6931}{10000} + \frac{249}{10000} + \frac{64}{3} \frac{6932}{10000} \frac{249/10000}{1 + 248/10000} < \frac{77}{500}. \end{aligned}$$

Together with $\frac{64}{25}(6931/10000)^2 > 6/5$ and (13), these are a finite rational certificate for the measured bound. The analytic separation follows already from $C_{\text{cl}} > 457/1000$.

For the more precise decimal value of E , the supplied evaluator applies (17) after power-of-two range reduction, with rational interval arithmetic throughout. It encloses u , evaluates $h_2(u)$, and checks the two endpoints stated in Theorem 1. In particular, combining the interval with (5) certifies $E - C_M > 0.005359$. This sharper decimal is supplementary to the rational gap $3/1000$.

References

- [1] K. M. R. Audenaert, J. Calsamiglia, Ll. Masanes, R. Muñoz-Tapia, A. Acín, E. Bagan and F. Verstraete, Discriminating states: The quantum Chernoff bound, *Physical Review Letters* **98**, 160501 (2007). [doi:10.1103/PhysRevLett.98.160501](https://doi.org/10.1103/PhysRevLett.98.160501).
- [2] M. Nussbaum and A. Szkoła, The Chernoff lower bound for symmetric quantum hypothesis testing, *The Annals of Statistics* **37**, 1040–1057 (2009). [doi:10.1214/08-AOS593](https://doi.org/10.1214/08-AOS593).
- [3] M. Hayashi, Discrimination of two channels by adaptive methods and its application to quantum system, *IEEE Transactions on Information Theory* **55**, 3807–3820 (2009). [arXiv:0804.0686](https://arxiv.org/abs/0804.0686).
- [4] J. Calsamiglia, J. I. de Vicente, R. Muñoz-Tapia and E. Bagan, Local discrimination of mixed states, *Physical Review Letters* **105**, 080504 (2010). [doi:10.1103/PhysRevLett.105.080504](https://doi.org/10.1103/PhysRevLett.105.080504).
- [5] W. Matthews and A. Winter, On the Chernoff distance for asymptotic LOCC discrimination of bipartite quantum states, [arXiv:0710.4113](https://arxiv.org/abs/0710.4113) (2007; revised 2008).
- [6] F. Salek, M. Hayashi and A. Winter, Usefulness of adaptive strategies in asymptotic quantum channel discrimination, *Physical Review A* **105**, 022419 (2022). [doi:10.1103/PhysRevA.105.022419](https://doi.org/10.1103/PhysRevA.105.022419).

- [7] M. Berta, O. Fawzi and M. Tomamichel, On variational expressions for quantum relative entropies, *Letters in Mathematical Physics* **107**, 2239–2265 (2017). [doi:10.1007/s11005-017-0990-7](https://doi.org/10.1007/s11005-017-0990-7).
- [8] QIQCOP Zoo, Chernoff exponents under separable and LOCC tests across copies, problem op_a370855db65d4d24, accessed 16 October 2026. https://qiqc-op.com/problem/op_a370855db65d4d24/.
- [9] T. Kassis, V. Agarwal, Y. He, D. Patel and A. M. Brueckner, Scientific Agent Skills: A Library of Procedural Knowledge for Research Agents, [arXiv:2609.00065](https://arxiv.org/abs/2609.00065) (2026).